The Effects of Risk on Self-Employment

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ABSTRACT. The effects of sectoral risk on the propensity to become self-employed are analysed. In contrast to previous studies which assume risk is concentrated only in the self-employment sector, a general equilibrium model is constructed in which uninsurable risk exists in *both* the paid- and self-employed sectors. It is shown that when agents wish to mix self-employment and employee participation, the effects of sectoral risk on the optimal self-employment decision is ambiguous. This result disappears if agents choose instead to participate in one sector alone.

1. Introduction

An important area of ongoing research in industrial economics is the analysis of the determination of self-employment. A resurgence of interest has followed the steady growth in self-employment in many industrial economies since the late 1970s, after decades of decline. In the U.K., self-employment increased by 57 per cent between 1979 and 1993, with nearly 3.2 million people being self-employed in 1993.[1] Several approaches have been developed in an effort to explain these trends, which emphasise to varying degrees economic, sociological, and psychological factors (see Keeble et al., 1993, for a review).

The salient economic approach views an individual's decision to enter paid- or self-employment as determined by relative returns offered in these sectors. Returns confer utility, and agents choose the sector which maximises expected utility (Lucas, 1978; Kihlstrom and Laffont, 1979; Jovanovic, 1982). This approach encapsulates forces such as 'unemployment push' and 'pull' factors (whereby unemployed workers are either pushed into self-employment or are pulled in by incentives on the supply side), since in both cases, workers are making a choice based on relative returns. Yet relative returns only tell one part of the story. It has been recognised for a long time – at least as far back as Knight (1921) – that self-employment returns are intrinsically riskier than returns to paid employment. Risk-averse agents will therefore demand greater expected returns in order to enter self-employment.

The purpose of this paper is to clarify the role of risk in the self-employment decision, and to examine the robustness of some assumptions routinely made in the modelling of self-employment in the presence of risky returns. Some studies assume that individuals are risk neutral (which implies that individuals are perfectly indifferent to any degree of wage volatility); and practially all studies in the literature to date assume that risk is concentrated in the self-employment sector only. The problem with artifical assumptions of this type is that they are unlikely to give robust predictions about the real effects of risk on self-employment choice. This is illustrated in this paper where a model is constructed which relaxes these assumptions. In particular, it is found that the ubiquitous simplifying assumption that risk is uniquely concentrated in the self-employment sector is not innocuous, as has been previously thought: relaxing it by subjecting paid as well as self-employment incomes to risk can have major implications for the predicted direction of sectoral risk on the self-employment decision.

The structure of the paper is as follows. Section 2 provides a brief summary of the literature as it relates to the effects of risk on the self-employment participation decision. Section 3 presents a model of optimal self-employment choice by heterogeneous risk averse agents in general equilibrium, under two different specifications of occupational choice. Section 4 concludes.

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2. Previous literature on risk

An obvious starting point for any analysis of risk and uncertainty on the decision to enter self-employment is Knight (1921). Knight considered the factors which influence entry into self-employment, and he stressed the special role of the entrepreneur as operating under uncertainty and bearing the risk of business failure. Knight went on to make an important distinction between risk and uncertainty. Risk describes a state of affairs where individuals may attach probabilities of occurrence to future events; uncertainty is a state of more profound ignorance where even the probabilities of events cannot be determined (Casson, 1982). Knight argued that in practice, uncertainty is endemic in business and permeates most business decisions.

Despite the early emphasis on uncertainty, few authors have followed this path.[2] It has been much more popular in the neoclassical literature to posit some stochastic influence on business outcomes, with a given structure which agents can observe or learn. In other words, it is risk which is more commonly modelled in practice. Stochastic forces have been modelled in several different ways to date. For example, Kihlstrom and Laffont (1979), Lippman and Rumelt (1982), Hopenhayn (1986), and Evans and Jovanovic (1989) introduce a random variable into the production function of the self-employed; Jovanovic (1982) incorporates random variation in the costs with which individuals can produce a given output; and Parker (1996a) models self-employment income as an Itô process, which is a dynamic process which evolves over time with both a deterministic and a stochastic component. In all cases, the first two moments of the stochastic element are known, or are learned (in the case of Jovanovic, 1982).

Common assumptions in all this work is that risk is (a) uninsurable, and (b) concentrated in the self-employed sector: wages in the paid employment sector are fixed and certain. Assumption (a) seems fairly realistic. Although Kihlstrom and Laffont (1982) raise the possibility of risk-diversification by issuing stock on the stock-market, this is not a feasible option for most of the self-employed (de Wit, 1993). Moreover, the market for accident, sickness and unemployment insurance for the self-employed is very limited in scope. However, assumption (b) does not seem at all realistic, and will be relaxed in this paper. A third assumption, (c), which is *not* common to all work in this area, is that agents are risk-averse. Kihlstrom and Laffont (1979) and Parker (1996a) both invoke this assumption. Using all three assumptions, it is easily shown that risk reduces the incentive to become self-employed, *ceteris paribus*. In the model of Kihlstrom and Laffont (1979), for example, it is demonstrated that, among individuals with different degrees of risk aversion, the least risk averse will choose self-employment over paid employment. In contrast, Parker (1996a) utilises the more common Lucas (1978) assumption that individual heterogeneity appears in the form of different managerial abilities. Individuals trade off higher risk with higher returns in self-employment; the most able are likeliest to become self-employed; and higher risk unambigiously reduces all individuals' propensities to become self-employed.

The assumption that agents are risk averse seems a reasonable one. However, some researchers remove it, and therefore also the 'downside' to volatility. This can generate some rather unrealistic predictions of market behaviour, as in Revelli and Tenga (1989), who assume costless entry and exit. With the 'downside' risk removed, greater volatility in business must *always* encourage entry from agents, since the 'upside' probability of high returns increases too. Risk neutrality does not therefore appear to be an attractive ingredient for a model of self-employment.

In the next section, a model based on risk-averse agents is developed which takes as a starting point a partial equilibrium analysis of self-employment developed by the author (Parker, 1996a). That analysis is generalised in two important respects: by allowing risk to enter also the paid employment sector; and by extending the model to general equilibrium.

3. The model

This section studies the effects of risk on self-employment and shows how the effects depend on how occupational choice is modelled. The material here is divided into three parts. In the first two, agents choose the optimal proportion of their worktime spent in self-employment and paid employment. In 3.1, the partial equilibrium case of a single agent is studied; this is extended to heterogeneous agents in general equilibrium in 3.2. In both 3.1 and 3.2, it is seen how sectoral risk has an ambiguous effect on the self-employment participation decision. In contrast, 3.3 has individuals choosing *either* self-employment *or* paid employment. It is seen that greater riskiness in a sector will always reduce the likelihood an agent chooses to participate in it.

3.1. *The single agent operating in partial equilibrium*

Following other models of intertemporal choice under uncertainty, including Dixit and Rob (1994), the basic self-employment model to be used here is framed in continuous time with an infinite planning horizon. Unlike Dixit and Rob, it will be assumed throughout the paper for simplicity that individuals do not incur switching costs when they adjust their participation in different occupations.[3] The problem commences with an individual agent seeking to maximise expected discounted intertemporal utility, subject to a dynamic stochastic budget constraint. Following the usual approach in the literature, the instantaneous utility function will be defined on income: $u = u(y)$, where $u(\cdot)$ is assumed to be concave with the usual properties; r is the individual's discount rate; and ε is the expectations operator. The objective is

$$\max \varepsilon \int_0^\infty u(y) e^{-rt} \mathrm{d}t. \qquad (1)$$

The individual can spend time either in self-employment, an activity denoted by S; or in paid employment or unemployment, E. Let w_i denote the expected per-period return from sector i, $i = E, S$; and let θ denote the proportion of time allocated to sector S. Normalising total working time to unity, $\varepsilon(y) = \theta w_S + (1 - \theta) w_E$. In Parker (1996a), the following assumptions were made to tie down the structure of the budget constraint further: (1) the individual's expected growth in returns, whose rate in sector i is $\dot{w}_i / w_i$ (the dot denoting a time derivative), is positively correlated with the individual's income. This is because greater income, whichever sector it is obtained from, facilitates greater investment in growth-promoting physical and human capital. Denote the growth-income correlation coefficients, which may vary between sectors, by λ_i. That is, $\dot{w}_i / w_i = \lambda_i y$, and so the expected budget constraint (i.e. its deterministic component) is

$$\varepsilon \mathrm{d}y = [w_S \lambda_S \theta + w_E \lambda_E (1 - \theta)] y \, \mathrm{d}t. \qquad (2)$$

The other assumptions were: (2) occupational risk and income are positively related;[4] and (3) that uninsurable risk is concentrated in the self-employment sector. For example, suppose that dz is the increment of a Wiener process z, which is a normally distributed random variable with a mean of zero. By assumption (2), its per-period variance will depend on y, and will be denoted $\varsigma^2(y)$. In the notation of the stochastic calculus, $\mathrm{d}w_S = \varsigma(y)\mathrm{d}z$ for the self-employed; by assumption (3), the absence of risk for employees would imply $\mathrm{d}w_E / \mathrm{d}z = 0$. It was further assumed for simplicity that $\varsigma(y)$ varied proportionately with y, i.e. $\varsigma(y) = \sigma y$, where σ is a parameter.

Although assumptions (1) and (2) seem reasonably innocuous, (3) needs to be relaxed since returns in neither sector are perfectly insurable. To generalise the model to allow for risk to be present in both sectors, suppose that when a shock dz hits the economy, both sectors are potentially affected, but that the variance in returns may differ between the sectors. That is, write $\mathrm{d}w_i = \varsigma_i(y)\mathrm{d}z = \sigma_i y \, \mathrm{d}z$ using the same parameterisation as above, where the σ_i are constants, which can be interpreted as risk parameters. By the chain rule, the stochastic component of the budget constraint is simply

$$\mathrm{d}y = [\theta \sigma_S + (1 - \theta) \sigma_E] y \, \mathrm{d}z. \qquad (3)$$

The total budget constraint is given by the sum of the deterministic (expected) component (2), and the 'disturbance' around it, (3):

$$\mathrm{d}y = [w_S \lambda_S \theta + w_E \lambda_E (1 - \theta)] y \, \mathrm{d}t + [\theta \sigma_S + (1 - \theta) \sigma_E] y \, \mathrm{d}z. \qquad (4)$$

The individual's problem is to choose their optimal desired commitment to self-employment, denoted by θ^*. This involves solving (1) subject to (4) and the initial condition $y(0) = y_0$ (given). A closed-form solution to this problem can be obtained for a very wide class of utility functions $u(y)$; but it suffices for present purposes to employ the simple and widely used special case of constant relative risk aversion.[5] Denote the Arrow-Pratt risk aversion parameter by γ: then it can be shown (Appendix 1) that the solution is

$$\theta^* = \frac{w_S\lambda_S - w_E\lambda_E}{\gamma(\sigma_S - \sigma_E)^2} - \frac{\sigma_E}{\sigma_S - \sigma_E} . \tag{5}$$

According to this solution, an individual will spend more time in self-employment the greater are growth-weighted returns in S relative to E, and the lower is their aversion to risk. *However, the effect on* θ^* *of the riskiness of returns in each sector,* σ_S and σ_E, *is ambiguous*. This is a very important, and perhaps surprising result. The intuition behind it will be explained below, when the exposition of the model is completed by extending the analysis to deal with heterogeneous agents in general equilibrium. It suffices here to point out that the above result is robust to other specifications of the utility function and to these extensions of the model. Incidentally, it can be seen that setting $\sigma_E = 0$ in (5) reduces the solution to the special case of Parker (1996a), where an increase in the riskiness of self-employment returns unambiguously reduces θ^*.

3.2. *Heterogeneous agents operating in general equilibrium*

It should be recognised that individuals differ in their endowments of managerial skills, which will induce variations in the relative returns individuals can realise.[6] Consider a person-specific parameter ξ_j, which takes potentially different values for each individual j in the workforce ($j = 1, \ldots, N$), giving rise to the following heterogeneous version of (5):

$$\theta^*_j = \xi_j \cdot \left(\frac{w^*_S\lambda^*_S - w^*_E\lambda^*_E}{\gamma(\sigma_S - \sigma_E)^2} \right) - \frac{\sigma_E}{\sigma_S - \sigma_E} . \tag{6}$$

Here, $(w^*_S\lambda^*_S - w^*_E\lambda^*_E)$ are the *average* relative returns prevailing amongst the workforce. Denote the density function of ξ by $f(\xi) \geq 0$: $\xi \in [\xi_L, \xi_U]$, where ξ_L and ξ_U define the support of the distribution. In reality, individuals typically enter *either* S or E: for example, less than 1 per cent of the workforce participate in both sectors simultaneously in the U.K. (Campbell and Daly, 1992: Table 27, p. 285). One modelling approach would be to define all individuals with $\theta^*_j \geq 0.5$ as 'self-employed', with the remainder being 'employees'; but this is not satisfactory if individuals in the latter category want to spend less time in employment than firms' minimum hours requirement for all workers. Denoting the (constant) normalised value of a firm's minimum hours stipulation by $\tilde{\theta}$ $(0 < \tilde{\theta} < 1)$, it seems preferable on these grounds to define a worker as an employee if $1 - \theta^*_j \geq \tilde{\theta}$, i.e. if $\theta^*_j \leq 1 - \tilde{\theta}$; and self-employed if $\theta^*_j > 1 - \tilde{\theta}$. Now let $F(\theta^*)$ denote the distribution function of the θ^* variable across the workforce. Then the number of self-employed individuals is simply $N^S = N[1 - F(1 - \tilde{\theta})]$. In order to find $F(1 - \tilde{\theta})$, use (6) to write, for any τ,

$$\begin{aligned} Pr[\theta_j = \tau] & \qquad\qquad\qquad\qquad\qquad\qquad (7) \\ &= Pr\left[\xi_j \cdot \left(\frac{w^*_S\lambda^*_S - w^*_E\lambda^*_E}{\gamma(\sigma_S - \sigma_E)^2} \right) - \frac{\sigma_E}{\sigma_S - \sigma_E} = \tau \right] \\ &= f\left(\frac{\gamma(\sigma_S - \sigma_E)^2[\tau + \sigma_E/(\sigma_S - \sigma_E)]}{w^*_S\lambda^*_S - w^*_E\lambda^*_E} \right), \end{aligned}$$

where the Jacobean of the transform is

$$f(\xi) = \left| \frac{w^*_S\lambda^*_S - w^*_E\lambda^*_E}{\gamma(\sigma_S - \sigma_E)^2} \right| \cdot Pr\left[\xi \left(\frac{w^*_S\lambda^*_S - w^*_E\lambda^*_E}{\gamma(\sigma_S - \sigma_E)^2} \right) - \left(\frac{\sigma_E}{\sigma_S - \sigma_E} \right) \right].$$

Then F is simply the integral of (7), so

$$\begin{aligned} N^S &= N[1 - F(1 - \tilde{\theta})] \qquad\qquad\qquad\qquad (8) \\ &= N\int_{1-\tilde{\theta}}^{1} f\left(\frac{\gamma(\sigma_S - \sigma_E)^2[\tau + \sigma_E/(\sigma_S - \sigma_E)]}{w^*_S\lambda^*_S - w^*_E\lambda^*_E} \right) d\tau. \end{aligned}$$

What is the structure of $f(\cdot)$? It seems likeliest that managerial talent in the workforce will be a scarce resource. If so, this would imply that the density function of the ability parameter, $f(\cdot)$, would be decreasing in ability. Accordingly, it will be assumed henceforth that $f'(\xi) < 0$, i.e. the frequency of people with managerial ability is less the greater that ability is specified to be. With equation (8), and with this assumption in place, the solution under general equilibrium can be analysed. The general equilibrium aspect of the problem involves recognising that an increase in the supply of individuals entering a sector will reduce the expected return in that sector. Denote the production function in sector i by $g_i(N^i)$, where N^i are the total number of members of sector i, as given by (8) above for N^S; note also that $N^E = N - N^S$.[7] As in Kihlstrom and Laffont (1979), assume non-increasing returns to scale: $g_i' > 0$, $g_i'' \leq 0$, for both i; and assume that factors are paid their marginal products: i.e.

$$w_i^* = p_i \cdot g_i'(N^i), \quad i = E, S \tag{9}$$

where p_i is the price of sector i's output. In the following, p_E will be normalised to unity. Substituting (9) into (8) yields the expression

$$N^S = N\int_{1-\tilde{\theta}}^{1} f\left(\frac{\gamma(\sigma_S - \sigma_E)^2[\tau + \sigma_E/(\sigma_S - \sigma_E)]}{p_S\lambda_S^* g_S'(N^S) - \lambda_E^* g_E'(N - N^S)}\right)\mathrm{d}\tau. \tag{10}$$

How do the parameters of the problem, chiefly γ, p_S, the λ_i^* and the σ_i, impact on aggregate self-employment? To answer this question, (10) must be implicitly differentiated. Denoting the derivative of N^S with respect to γ by N_γ^S, etc., the following results are obtained:

$$\begin{aligned} &N_\gamma^S < 0, \quad N_{\lambda_E}^S \leq 0, \quad N_{\lambda_S}^S \geq 0, \\ &N_{p_S}^S \geq 0, \quad N_{\sigma_E}^S \gtreqless 0, \quad N_{\sigma_S}^S \gtreqless 0 \end{aligned} \tag{11}$$

(see Appendix 2). These results state that aggregate self-employment decreases with the degree of risk aversion and is non-increasing in the expected growth factor relating to incomes in sector E; self-employment is non-decreasing in the relative output price of, and the expected growth factor relating to incomes in, sector S. However, the effects of riskiness of returns, when it is present in both sectors, *remains ambiguous as found in the partial equilibrium analysis earlier*. There, a change in sectoral risk changed the optimal θ^* for an individual agent; here, such a change alters the probability of having a θ_j^* which is greater than $1 - \tilde{\theta}$, and so alters aggregate self-employment.

What is the intuition for the ambiguity of sectoral risk on self-employment participation? The answer lies in the fact that there will in general be some covariance between the returns which make up the two-sector labour 'portfolio' of the agent. The *total* riskiness of the agent's portfolio is therefore not a monotonic decreasing function of each of the sectoral risks: it is instead a convex combination of them. It is the overlap between the individual sectoral risks in this context which provides the ambiguity of sectoral risk on self-employment participation. This problem is in fact directly analagous to the ambiguity of the riskiness of individual assets on total asset portfolio choice routinely taught in undergraduate finance courses (see e.g. Brealey and Myers, 1992). Notice from (6) that the smaller is θ_j^*, i.e. the more time the agent wishes to spend in paid employment, the greater will be the 'perverse' effect of rising self-employment riskiness encouraging agents to spend more time in self-employment. This is because the 'perverse' effect comes from the second (negative) term on the right-hand side of (6), which can only dominate when the first term (and hence θ_j^*) is small.

3.3. *Agents who choose one sector only*

So far, individuals have chosen the mix of time they would like to spend in each sector. However, in this sub-section it will be shown that if an agent *must* choose either to work full-time in self-employment or full-time in regular employment, the ambiguous effects of sectoral risk on sectoral participation disappear.

Adapting the objective function and Itô process in 3.1 to the current problem gives the following decision problem:

$$\max_{i \in \Omega} \ \varepsilon\int_0^\infty u(y_i)e^{-rt}\mathrm{d}t \quad \Omega = \{S, E\} \tag{12}$$

$$\text{s.t.} \quad \mathrm{d}y_i = \alpha_i y_i \mathrm{d}t + \sigma_i y_i \mathrm{d}z \quad i \in \Omega \tag{13}$$

where α_S, $\alpha_E \in \mathbf{R}$ are expected income growth rates in each sector; where $y_i(0) = y_0$ for both i; and where it is assumed as in 3.1 and 3.2 that there are no switching costs between the two sectors. Notice here that the agent is *choosing i*. Costless switching between sectors means that a switcher is compensated with the same income at the time of the switch. This fact is used in Appendix 3 to prove that with constant relative risk aversion, the agent's choice problem solves

$$\max_{i \in \Omega} \ \{2\alpha_i - \sigma_i^2\gamma\}. \tag{14}$$

It is obvious from inspection of (14) that, *ceteris paribus*, greater risk σ_i^2 in a sector *unambiguously* reduces the strength of the agent's preference to choose it. Moreover, this result carries through for heterogeneous agents and in general equilibrium. This can be easily verified by applying the same techniques of sub-section 3.2 to (14); the proof is excluded here for brevity.

6. Conclusions

This paper has examined the effects of risk on aggregate self-employment. A general equilibrium model was constructed, with risk-averse optimising agents who face uninsurable risk in both the paid- and self-employment sectors, but who do not incur adjustment costs when switching sectors. It was shown that when agents wish to mix self-employment and employee participation, the effects of sectoral risk on the optimal self-employment decision is ambiguous. In particular, an increase in the riskiness of self-employment incomes may actually increase agents' optimal participation in self-employment. This counter-intuitive finding is explained by observing that agents can sometimes reduce risk by taking a convex combination of two risky alternatives. Any *covariance* of returns in the two sectors is sufficient to generate this result.

It was also shown that if agents choose instead to participate in one sector alone, this result disappears. In this case, sectoral risk has unambiguous effects on self-employment: greater riskiness of incomes in a sector will always reduce the likelihood an agent chooses to participate in it. The effects of risk on self-employment therefore hinge on whether agents choose an optimal hourly mix and then a sector, or just a sector. This is essentially an empirical issue, which remains uninvestigated to the author's knowledge. At any rate, our results here demonstrate that risk in *both* sectors needs to be modelled in order to understand self-employment choice. Empirical work which recognises this point is already underway.[8]

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Appendix 1: Proof of equation (5)

Writing $h_i \equiv w_i\lambda_i$, $i = S, E$, and using the approach of dynamic programming, the value function for this problem (i.e. the maximum expected value available to the individual from remaining income) is:

$$rV(y) = \max_{\theta} \left\{ u(y) + V'(y)[h_E + \theta(h_S - h_E)]y + \frac{1}{2} y^2 V''(y)[\theta\sigma_S + (1-\theta)\sigma_E]^2 \right\}.$$

The first order condition is therefore

$$V'(y)(h_S - h_E)y + y^2V''(y)[\sigma_E(\sigma_S - \sigma_E) + \theta(\sigma_S - \sigma_E)^2] = 0,$$

which implies that the optimal θ is

$$\theta^* = -\left[\frac{V'(y)(h_S - h_E)}{V''(y)y(\sigma_S - \sigma_E)^2}\right] - \frac{\sigma_E}{\sigma_S - \sigma_E}.$$

Putting this back into the value function and simplifying, we obtain

$$rV(y) = u(y) + \phi_1 V'(y)y + \phi_2\left\{\frac{[V'(y)]^2}{V''(y)}\right\} + \phi_3 y^2 V''(y),$$

where

$$\phi_1 = h_E - \frac{3\sigma_E(h_S - h_E)}{(\sigma_S - \sigma_E)}$$

$$\phi_2 = -\frac{3}{2}\left[\frac{h_S - h_E}{\sigma_S - \sigma_E}\right]^2$$

$$\phi_3 = -\frac{\sigma_E^2[1 + (\sigma_S - \sigma_E)^2]}{2}$$

are constants. The problem is to solve for the functional form of $V(y)$ and so to derive a complete solution for θ. For the CRRA utility function, $u(y) = y^{1-\gamma}/(1-\gamma)$, where γ is the coefficient of relative risk aversion. Putting this into the above expression, a solution presents itself in the form of

$$V(y) = Ay^{1-\gamma},$$

where A is some constant. Now noting that $V'(y) = A(1-\gamma)y^{-\gamma}$ and also that $V''(y) = -A\gamma(1-\gamma)y^{-(\gamma+1)}$, then θ^* becomes

$$\theta^* = \left[\frac{h_S - h_E}{\gamma(\sigma_S - \sigma_E)^2} - \frac{\sigma_E}{\sigma_S - \sigma_E}\right] = \frac{w_S\lambda_S - w_E\lambda_E}{\gamma(\sigma_S - \sigma_E)^2} - \frac{\sigma_E}{\sigma_S - \sigma_E}$$

as required. It should be borne in mind that this is an interior solution; 'extreme' values of θ^* are taken to be zero or unity as appropriate (see e.g. Merton, 1969, for a similar approach to a problem of this type).

Appendix 2: Proof of the derivatives in (11)

First define the set of parameters of interest by Ω: $\Omega = \{\gamma, \lambda_S^*, \lambda_E^*, p_S, \sigma_S, \sigma_E\} = \{\omega_1, \omega_2, \ldots, \omega_6\}$. Next define $f(\Omega, \tau)$ as the function $f(\cdot)$ under the integral sign in (10):

$$f(\Omega, \tau) \equiv f\left(\frac{\gamma(\sigma_S - \sigma_E)^2[\tau + \sigma_E/(\sigma_S - \sigma_E)]}{p_S\lambda_S^* g_S'(N^S) - \lambda_E^* g_E'(N - N^S)}\right) = f(\xi). \tag{15}$$

To do the implicit differentiation, first define a function G such that $G(N^S, \omega_k) = 0$, $\forall_k$: i.e.

$$G = N^S - N\int_{1-\tilde{\theta}}^{1} f(\Omega, \tau)d\tau = 0. \tag{16}$$

Then the derivatives of interst are

$$\frac{dN^S}{d\omega_k} = -\frac{G'_{\omega_k}}{G'_{N^S}} \qquad \forall_k. \tag{17}$$

The partial derivative G'_{N^S} is examined first. Using (15), and writing $h_i^* \equiv w_i\lambda_i^*$, differentiate (16):

$$G'_{N^S} = 1 - N\int_{1-\tilde{\theta}}^{1} \left(\frac{\partial f(\Omega, \tau)}{\partial(h_S^* - h_E^*)} \cdot \frac{\partial(h_S^* - h_E^*)}{\partial N^S} \right) d\tau \tag{18}$$

$$= 1 - N[\lambda_S^* p_S g_S''(N^S) + \lambda_E^* g_E''(N^E)] \int_{1-\tilde{\theta}}^{1} \frac{\partial f(\Omega, \tau)}{\partial(h_S^* - h_E^*)} d\tau$$

The derivative under the integral sign in (16) can be re-written as

$$\frac{\partial f(\Omega, \tau)}{\partial(h_S^* - h_E^*)} = \frac{\partial f(\xi)}{\partial \xi} \cdot \frac{\partial \xi}{\partial(h_S^* - h_E^*)} \tag{19}$$

By the assumption of scarcity of managerial ability stated earlier, the first term of the product on the RHS of (19) is negative for all τ (strictly speaking, the weaker condition $f'_\xi(\xi) < 0: \xi \in [1 - \tilde{\theta}, 1]$ is all that is required, rather than for all $\xi \in [\xi_L, \xi_U]$). Since the *denominator* of (15) is $(h_S^* - h_E^*)$, the second term of this product is also negative for all τ. It therefore follows that the derivative (19) is positive for all (relevant) τ, as must be the integral in (18). What of the rest of (18)? The term in square brackets of that equation must be non-positive, since by non-increasing returns to scale $g_i'' \leq 0$, $i = E, S$ (note that the growth factors and output prices are positive). Bringing all this information together, it therefore follows that $G'_{N^S} > 0$.

The derivatives forming the numerator of (17) are now examined. From (16)

$$-G'_{\omega_k} = N\int_{\tilde{\theta}}^{1} \frac{\partial f(\Omega, \tau)}{\partial \omega_k} d\tau, \qquad \forall_k.$$

As above, the derivative under the integral sign can be split up into a product of two components, where the first is negative for all (relevant) τ, reflecting scarce managerial talent; and where the second is $\partial\xi/\partial\omega_k$. By (15), the latter derivative ($\forall\tau$) is positive for γ and non-negative for λ_E^*; is non-positive for λ_S^* and p_S; and ambiguous for σ_S and σ_E. Hence $-G'_\gamma < 0$; $-G'_{\lambda_E^*} \leq 0$; $-G'_{\lambda_S^*}, -G'_{p_S} \geq 0$; and $-G'_{\sigma_S}, -G'_{\sigma_E}$ are ambiguous. Since these derivatives are divided by a positive number G'_{N^S}, the signs of $dN^S/d\omega_k$ are unchanged and appear as in (11) in the text.

Appendix 3: Proof of equation (14)

Applying Itô's lemma to $u(y_i)$ where the y_i evolve according to (13),

$$du = \left[\frac{\partial u}{\partial t} + \alpha_i y_i \frac{\partial u}{\partial y_i} + \frac{1}{2}\sigma_i^2 y_i^2 \frac{\partial^2 u}{\partial y_i^2} \right] + \sigma_i y_i \left(\frac{\partial u}{\partial y_i} \right) dz$$

$$= \left[\alpha_i y_i^{1-\gamma} - \frac{1}{2}\sigma_i^2 \gamma y_i^{1-\gamma} \right] dt + \sigma_i y_i^{1-\gamma} dz$$

$$= \left[(1-\gamma) \left(\alpha_i - \frac{1}{2}\sigma_i^2\gamma \right) \right] u\, dt + \sigma_i(1-\gamma)u\, dz$$

which follows Brownian motion. Since $\varepsilon[dz] = 0$,

$$\varepsilon[u(y_i)] = u(y_0) \cdot \exp\left\{ (1-\gamma)(\alpha_i - \frac{\sigma_i^2\gamma}{2})t \right\}.$$

Substituting this expression for the utility function into (12) yields

$$\max_{i \in \Omega} \left\{ \frac{y_0^{1-\gamma}}{1-\gamma} \left[r + (1-\gamma)\left(\frac{\sigma_i^2\gamma}{2} - \alpha_i \right) \right]^{-1} \right\}, \tag{20}$$

provided that the denominator of (20) in square brackets is positive. Equation (14) then follows directly from (20), for all $\gamma > 0$.

Notes

1 The 1993 figure is down from the peak of over 3.5 million registered in 1990. Source: *Economic Trends Annual Supplement*, 1995, HMSO.

2 Blanchflower and Oswald (1991) are an exception. Despite its title, the author's (1996a) paper also deals with risk, not uncertainty in the Knightian sense.

3 This extension could be incorporated fairly easily into the current sub-section without changing the results; but would cause intractability for the analysis of heterogeneous agents in 3.2. It is therefore excluded here.

4 For example, this is true when comparing sectors E and *S*: inequality *and* average incomes are greater in the latter than in the former (see e.g. Table 9.1 of Meager et al, 1996).

5 For example, the problem can be solved using HARA (Hyperbolic Absolute Risk Aversion) preferences, which nests most of the best known utility functions as special cases, including constant relative risk aversion, used here.

6 This is a common assumption in the literature (de Wit, 1993), although some authors choose instead to introduce heterogeneity through variations in risk aversion preferences (e.g. Kihlstrom and Laffont, 1979); that approach is not followed here.

7 It does not matter here who owns the technology described by these production functions. What is worth noting however is that workers in this model are not necessarily employees of the self-employed, as they are constrained to be in the model of Kihlstrom and Laffont (1979). That feature of the Kihlstrom and Laffont model does not seem to be at all realistic, since in the U.K., for example, less than one per cent of the workforce was employed by self-employed people in 1991 (source: Campbell and Daly, 1992, p. 285). A more realistic alternative is that ownership of capital is dispersed throughout the economy, and does not affect the sectoral choice decision of the overwhelming majority of individuals.

8 See e.g. Foti and Vivarelli (1994) and current work by the author (Parker, 1996b).

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